# A Three Phase Current Regulation Strategy with Inductor over Saturation Region

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Abstract—This paper presents an improved current regulating strategy with a feed-forward term in stationary reference frame to compensate widely varying inductance of three phase AC inductor over magnetic saturation region. In conventional current regulators such as state decoupled PI controller and complex vector PI controller, inductor in three phase circuit is presumed as linear and constant, and always operating far lower than the magnetic saturation level region. However, because of three phase instantaneous current difference and nonlinear characteristics of magnetic material, inductance of each phase would not be the same among three phase and would be time varying. Moreover, because of saturation characteristics, inductors are oversized to avoid the saturation region. With the proposed control strategy by extracted self and mutual inductance look-up table, not only inductance variation can be compensated, but also low order harmonics, especially 3rd order harmonics, are considerably reduced as well when operating over saturation current level. Experiment results are presented to verify the validation of the proposed control strategy.

Keywords—current regulator; varying inductance; magenetic saturation; feed-forward control

## I. INTRODUCTION

For high performance control of three phase ac circuit, current regulation is usually used as inner loop of three phase converter control system. The most widely used current regulators for three phase ac circuit with a PWM converter are the proportional plus integral (PI) current regulator on the synchronous reference frame with state decoupling terms and the complex vector current regulator on the frame, due to their superior current regulation capability for wide frequency range [1]-[4]. In these two regulators the inductance parameters are both assumed to be constant with only self-inductance and have linear characteristics. However, in order to achieve better performance of current regulation, modelling and prediction of circuit parameters such as inductor are needed. In three phase circuit, inductance values of three phase are varying instantaneously. The three phase inductor is usually made by single three limb EI core, and each phase inductor is coupled

to others. Moreover, because of the cost and size issue the magnetic saturation level of the inductor is near or sometimes lower than the rated current of the converter. Therefore, the inductor model used in PI and complex vector regulators would be not specific and accurate.

Wide inductance variation was considered in [5] for digital control of three phase bidirectional inverter and succeeded in regulating over saturation level current of single core inductor. However, control algorithm combined with SVPWM duty calculation made regulation become complicated and might not be easily implemented with low cost DSP. Moreover, it is not suitable for all PWM methods and mutual flux of EI core type inductor had not been considered.

This paper presents an improved current regulation strategy with a feed-forward term in stationary reference frame to compensate widely varying inductance of three phase AC inductor over saturation level. Conventional current regulators are reviewed in section II as a background of the understanding of ac current regulator. The proposed current regulator is based on the conventional ones and has some enhancement. Section III specifically presents the improved feed-forward control algorithm and the method of parameter table extraction and mutual inductance calculation. Experiment results are presented in section IV to verify the validation of the proposed current controller.

## II. REVIEW OF CONVENTIONAL AC CURRENT REGULATORS

Conventional PI regulator works in synchronous reference frame controlling d and q axis current simultaneously with proportional and integral gains, which are set as the function of regulation bandwidth and parameter of ac system [4]. Due to the rotation of the reference frame, PI regulator achieves decoupling by feed-forward term in synchronous reference

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frame not only contains back EMF in d-q axis but also terms

such as 
$$-\omega_e \hat{L}_{qs}^e$$
 in d axis and  $+\omega_e \hat{L}_{ds}^e$  in q axis as follows:

S

$$V_{ds\_ff}^e = -\omega_e Li_{qs}^e + e_{ds} \quad , \tag{1}$$

$$V_{qs\_ff}^{e} = +\omega_e \hat{L} i_{ds}^{e} + \hat{e}_{qs}^{e} \quad .$$
<sup>(2)</sup>

Complex vector regulator improves decoupling issue by moving the term from feed-forwarding term to integral term and reduces dependency on parameter error of inductance variation which cannot be compensated in PI regulator. In the complex vector regulator after transforming equation from (3) to (4), extra integral term is added to reduce feed-forward term in synchronous reference frame only including back EMF component. This makes the complex vector regulator be robust to parameter error.

$$V_{dq}^{s} = Ri_{dq}^{s} + Lpi_{dq}^{s} , \qquad (3)$$

$$V_{dq}^{e} = Ri_{dq}^{e} + (L + j\omega_{e})i_{dq}^{s} \quad .$$
<sup>(4)</sup>

#### III. VARYING INDUCTANCE FEED-FORWARD CURRENT CONTROL

The proposed feed-forward current control for three phase varying inductance over saturation level is derived by two parts: control algorithm and parameter table extraction.



Fig. 1. Three phase R-L-EMF circuit with varying inductance

## A. Feed-Forward Control Strategy in Stationary Reference Frame

Inductors in three phase AC balanced circuit are usually assumed to have linear characteristic and without mutual inductance from other phases. Based on this assumption, inductance parameters in conventional current regulators are set as constant values. However, inductors are time-varying in all three phases, and mutual inductances between phases also exist and are varying. This situation is even severer when current rises up to over the magnetic saturation level of the inductor. And, three phase R-L-EMF circuit with varying inductance can be depicted as shown in Fig.1. It is assumed that instantaneous sum of three phase EMF voltages,  $e_{abcs}$ , is zero.

This circuit can be described as follows:

$$\mathbf{V}_{\mathbf{abcs}} = R \cdot \mathbf{i}_{\mathbf{abcs}} + p \lambda_{\mathbf{abcs}} + \mathbf{e}_{\mathbf{abcs}} \ . \tag{5}$$

, where p is the derivative operator, and

$$\mathbf{V_{abcs}} = \begin{bmatrix} V_{as} & V_{bs} & V_{cs} \end{bmatrix}^T , \tag{6}$$

$$\mathbf{i}_{abcs} = \begin{bmatrix} i_{as} & i_{bs} & i_{cs} \end{bmatrix}^T , \tag{7}$$

$$\mathbf{e}_{\mathbf{abcs}} = \begin{bmatrix} e_{as} & e_{bs} & e_{cs} \end{bmatrix}^T , \tag{8}$$

and 
$$\lambda_{abcs} = \begin{bmatrix} \lambda_{as} & \lambda_{bs} & \lambda_{cs} \end{bmatrix}^T$$
 . (9)

Because of unbalanced and mutual inductances, flux linkage can be presented as below:

$$\boldsymbol{\lambda_{abcs}} = \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{as} & L_{ab} & L_{ac} \\ L_{ba} & L_{bs} & L_{bc} \\ L_{ca} & L_{cb} & L_{cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$
(10)

, where  $L_{xs}$  and  $L_{xy}$  (x, y = a, b, c) are self and mutual inductance of each phase. Particularly, every inductance value is different and varying. But, by the reciprocity,  $L_{xy}=L_{yx}$ 

In order to simplify control algorithm caused by varying inductance, matrix of inductance can be divided into two parts shown in (11),

$$\mathbf{L_{abcs}} = \begin{bmatrix} L_{as} & L_{ab} & L_{ac} \\ L_{ba} & L_{ts} & L_{bc} \\ L_{ca} & L_{cb} & L_{cs} \end{bmatrix} = \begin{bmatrix} L_{as} & 0 & 0 \\ 0 & L_{bs} & 0 \\ 0 & 0 & L_{ts} \end{bmatrix} + \begin{bmatrix} L_{as} - L_{s} & L_{ab} & L_{ac} \\ L_{ba} & L_{bs} - L_{s} & L_{bc} \\ L_{ca} & L_{cb} & L_{cs} - L_{s} \end{bmatrix}$$
(11)

, where Ls is the constant inductance parameter used in conventional PI and complex vector current regulators. And

Thus,  $V_{abcs}$  can be derived as follows:

$$\mathbf{V}_{\mathbf{abcs}} = R \cdot \mathbf{i}_{\mathbf{abcs}} + L_s \cdot p \mathbf{i}_{\mathbf{abcs}} + \mathbf{e}_{\mathbf{abcs}} + p (\mathbf{L}_{\mathbf{abcs}}^s \cdot \mathbf{i}_{\mathbf{abcs}})$$
(12)

, where  $V_{abcs}^{c}$  is exactly the same with conventional PI regulator and it can be implemented in synchronous reference frame by a conventional PI regulator.

Due to the fact that  $V^{s}_{abcs}$  is the derivative of product of two varying vectors regarding time, the transformation of this

term into synchronous frame is complicated and time consuming. And, a coupled feed-forward term in stationary reference frame is presented in this paper. Control strategy with the proposed feed-forward term in stationary reference d-q frame is shown in Fig.2, and transformation to stationary frame can be done by (13) and (14).



Fig. 2. Control algorithm with the proposed feed-forward term in stationary reference frame

$$V_{ds}^{s} = \frac{2}{3} \left\{ \frac{d}{dt} \left[ L_{as} - L_{s} - \frac{1}{2} L_{ba} - \frac{1}{2} L_{ca} \right] i_{as} + \left[ L_{as} - L_{s} - \frac{1}{2} L_{ba} - \frac{1}{2} L_{ca} \right] \frac{d}{dt} i_{as} \right. \\ \left. + \frac{d}{dt} \left[ L_{ab} - \frac{1}{2} L_{bs} + \frac{1}{2} L_{s} - \frac{1}{2} L_{cb} \right] i_{bs} + \left[ L_{ab} - \frac{1}{2} L_{bs} + \frac{1}{2} L_{s} - \frac{1}{2} L_{cb} \right] \frac{d}{dt} i_{bs} \right. \\ \left. + \frac{d}{dt} \left[ L_{ac} - \frac{1}{2} L_{bc} - \frac{1}{2} L_{cs} + \frac{1}{2} L_{s} \right] i_{cs} + \left[ L_{ac} - \frac{1}{2} L_{bc} - \frac{1}{2} L_{cs} + \frac{1}{2} L_{s} \right] \frac{d}{dt} i_{cs} \right\} \\ \left. \cdot (13) \right\}$$

$$V_{qs}^{s} = \frac{1}{\sqrt{3}} \left\{ \frac{d}{dt} [L_{ba} - L_{ca}] i_{as} + [L_{ba} - L_{ca}] \frac{d}{dt} i_{as} + \frac{d}{dt} [L_{bs} - L_{s} - L_{cb}] i_{bs} + [L_{bs} - L_{s} - L_{cb}] \frac{d}{dt} i_{bs} + \frac{d}{dt} [L_{bc} - L_{cs} + L_{s}] i_{cs} + [L_{bc} - L_{cs} + L_{s}] \frac{d}{dt} i_{cs} \right\}$$
(14)

#### B. Look-up Table Feed-Forward Strategy

From Part A, varying values of self and mutual inductance are needed for the proposed stationary reference frame feedforward terms. According to the nonlinear B-H characteristic of inductor, a look-up table between current and inductance including mutual flux could be acquired through off-line test of the inductor.

A three phase inductor can be modeled as a magnetic equivalent circuit with magneto-motive force (MMF), flux and magnetic reluctance, which can be looked upon as analogous

to an electric circuit [6]. Equivalent circuit based on above description is shown in Fig.3.



Fig. 3. EI core inductor equivalent circuit



Fig. 4. DC bias table at 10 kHz from 0-10A

A DC bias table of an inductor under consideration at 10 kHz from 0A to 10A is shown in Fig.4, where the relationship between input winding current and corresponding dynamic inductance can be acquired. Therefore, from (15)-(18), flux linkage can be integrated by dynamic inductance, and corresponding flux and magneto-motive force can be calculated. Dynamic permeance, which is inverse of reluctance, is the differential of flux by MMF. Therefore, table regarding current, self-inductance, deduced self-flux, and self-dynamic permeance can be obtained as follows.

$$L_{dyn} = \frac{d\lambda}{dt}$$
, (15)  $\phi = \frac{\lambda}{N}$ , (16)

$$F = N \cdot I$$
, (17)  $P_{diff} = \frac{d\phi}{dF}$ . (18)

Mutual flux comes from other limbs of inductor. After self flux and dynamic permeance calculation above, self-flux would be divided to become mutual flux of the other two limbs. The magnitude of mutual flux is decided by the dynamic permeance value at that moment as (19).

$$\phi_{mutual\_center\_to\_left} = \phi_{self\_center} \cdot \left(\frac{P_{diff\_left}}{P_{diff\_left} + P_{diff\_right}}\right).$$
(19)

Total flux of one limb is the sum of self and mutual flux. And referring to the table by mutual flux, mutual inductance could be evaluated. Fig.5 shows the example of look-up table usage.



Fig.5. Example of look-up table usage in proposed table feed-forward strategy

#### C. Estimated Inductor Voltage Feed-Forward Strategy

In the experiment, the line to line voltage of the grid and the phase current are measured between inductor and the grid. And, the three phase voltages applied to the inductor are given by PWM voltage references by DSP signals. Thus, after transforming from line to line voltage to phase voltage, the phase voltage differences of grid voltages and PWM voltage references are inductor voltages of each limb. From (5) and (10), inductor voltage of each phase can be presented by (20)-(22) as follows:

$$V_{la} = \frac{d}{dt} \left( L_{as} i_{as} + L_{ab} i_{bs} + L_{ac} i_{cs} \right) , \qquad (20)$$

$$V_{lb} = \frac{d}{dt} \left( L_{ba} i_{as} + L_{bs} i_{bs} + L_{bc} i_{cs} \right) , \qquad (21)$$

$$V_{lc} = \frac{d}{dt} \left( L_{ca} i_{as} + L_{cb} i_{bs} + L_{cs} i_{cs} \right)$$
(22)

, where  $V_{la}$ ,  $V_{lb}$  and  $V_{lc}$  are inductor voltage of each phase and limb.

In this case, feed-forward voltage terms, (13) and (14), in dq axis of stationary reference frame can be easily substituted by inductor voltages by (20)-(22). And, (23) and (24) can be derived.

$$V_{ds}^{s} = \frac{2}{3} \left( V_{la} - L_{s} \frac{d}{dt} i_{as} -\frac{1}{2} V_{lb} + \frac{1}{2} L_{s} \frac{d}{dt} i_{bs} , \qquad (23) -\frac{1}{2} V_{lc} + \frac{1}{2} L_{s} \frac{d}{dt} i_{cs} \right)$$

$$V_{qs}^{s} = \frac{1}{\sqrt{3}} \left( V_{lb} - L_{s} \frac{d}{dt} i_{bs} -V_{lc} + L_{s} \frac{d}{dt} i_{cs} \right) . \qquad (24)$$

Therefore, instead of using table parameters of self and mutual inductances, feed-forward terms are calculated by sensing voltages and PWM signals, which significantly simplify the implementation of the current regulator.

### IV. EXPERIMENTAL RESULT

To verify the performance of the proposed feed-forward current control strategy in magnetic saturation level, experiments among 4 regulators, namely conventional state decoupled synchronous reference frame PI regulator, complex vector PI regulator, the proposed regulator with look up table, and the proposed regulator with estimated inductor voltage, are compared. The parameters of the experimental system are listed in Table 1.

TABLE 1. Experimental system parameters

Quantity	Values
AC grid voltage;	110 Vrms
line to line voltage	
DC link voltage	200 V
Fundamental frequency	60 Hz
PWM switching frequency	5 kHz
Ls for Kp (P gain of regulator)	400 uH
Regulator bandwidth	2*PI*500
Dead time in digital control	3 us
Inductor saturation level	< 0.3 A

Experiment system is based on 200V DC link voltage and 5 kHz switching frequency PWM converter connected to three phase AC grid, whose voltage is 110Vrms line to line. System is shown in Fig.6.



Fig. 6. Experimental system for verification of the proposed regulator



Fig. 7. Experiment results: phase current waveform and FFT plots of 4 regulators at 5A

Fig.7 shows the current waveforms and FFT plots of conventional and the proposed current regulators at 5A q-axis reference (more than 10 times of saturation level). The corresponding FFT plot from 0 to 600Hz including fundamental current at 60Hz and 3rd, 5th and 7th current harmonics are shown in the figure. It can be seen that when current goes up to the saturation level, conventional PI and complex vector current regulator cannot compensate the inductance difference from non-saturation value. And, the current waveforms become highly distorted and have large low frequency harmonics, especially 3<sup>rd</sup> order harmonics. However, when using the proposed feed-forward current regulation strategies, the inductance difference due to magnetic saturation and mutual flux has been compensated by proper varying inductance provided by table feed-forward term or estimated inductor voltage feed-forward term. From the figure, it can be seen that the low frequency harmonics are reduced significantly. Percentages of 3<sup>rd</sup> order harmonics are also shown in Fig.7 which are inside of each FFT plot. The

proposed two types of regulators reveal much better performance in reducing 3rd order harmonics. Because of less parameter error, the proposed regulator with estimated inductor voltage works even better than the proposed one with It reduce the 3<sup>rd</sup> harmonics by 49% further compared to the regulator with the table.

Fig.8 shows the experimental current waveform and FFT results of 4 regulators at 8A current reference. Comparing with conventional current regulators, the proposed two regulator still have much better current controllability even in such highly saturated condition of the inductor (more than 20 times). However performance of proposed regulators under 8A are worse than under 5A, and the performance of regulator with the estimated inductor voltage is rapidly deteriorated. Percentages of 3<sup>rd</sup> order harmonics are also shown within Fig.8 FFT plots. From the figures, it can be seen that the proposed regulator can reduce the 3<sup>rd</sup> harmonics by 37% and 52% respectively compared to the conventional state decoupled PI regulator.



Fig. 8. Experiment results: phase current waveform and FFT plots of 4 regulators at 8A

## V. CONCLUSION

This paper presents an improved current regulating strategy with a feed-forward term in stationary reference frame to compensate widely varying inductance of three phase AC inductor over magnetic saturation region.

By the proposed table feed-forward control strategy, varying inductance extraction and mutual flux calculation method, circuit phase current can be controlled up to over higher magnetic saturation level. The feasibility of the strategy and parameter table accuracy have been validated by experimental results at different current reference levels. And the proposed feed-forward compensation strategy has reduced low order harmonics by 60% compared to the conventional current regulators at a certain operating point.

In addition, an estimation method of the inductor voltage in conjunction with the proposed regulator has been presented. It makes the implementation of the proposed regulator be simple. In the method, without any additional experiment equipment and off line test, only by grid voltage, measured current, and PWM voltage reference, the proposed regulator can be implemented. And the performance of this regulator has been also validated by experiment at different current reference levels. And it can be said that the regulator with estimated inductor voltage can reduce the 3<sup>rd</sup> harmonics by at least 20% more compared to the proposed regulator with the look up table.

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