# An Enhanced Sensorless Control Method for PMSM in Rapid Accelerating Operation

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Abstract--This paper presents an enhanced sensorless method to estimate rotor position for PMSM. The proposed method is based on the estimation of the back-EMF using a state observer. The conventional sensorless methods based on back-EMF estimation has problem of model discordance to the real motor model in electrical or mechanical dynamic state. An analysis is performed to verify how this problem deteriorates the estimation performance in the dynamic situations. Proposed method modifies state observer to match to the real motor model even in a fast accelerating situation. In order to verify feasibility of the proposed method, experimental results in a steady state and a dynamic state are presented.

*Index Terms*— Rapid acceleration, Sensorless, SMPMSM, State observer

#### I. INTRODUCTION

The Surface Mounted Permanent Magnet Synchronous Machine (SMPMSM) has a lot of attractive characteristics such as high efficiency and high torque density. Therefore, it is widely used in various industrial applications. In order to drive the PMSM efficiently, it is demanded to obtain the accurate rotor position information. The rotor position can be detected by using a position sensor such as optical encoders or resolvers. However these position sensors increase the system volume and cost and reduce the reliability of the system. Because of these reasons, many studies have been reported to acquire the position information without additional position sensors [1]-[7].

Generally, there are two types of main sensorless strategies. One is using the saliency with high frequency signal injection [2] and the other is using the back electromotive force (EMF) estimation with fundamental excitation [1]-[7]. The former approach can be applied in low speed range and even at standstill. But this method requires additional signal for position estimation. The latter strategy is difficult to guarantee its performance at low speed. However, this technique is relatively simple to implement and it is unnecessary to have high frequency signals which might be the source of acoustic noise. In high speed applications, the latter strategy is widely used due to the above reasons, and this paper focuses on how to estimate the back-EMF for the sensorless control of PMSM.

Various methods have been investigated to estimate the back-EMF. This category can be divided into two subcategories. One is using stator voltage equations directly [1]-[3] and the other is using a form of state observer [4]-[7]. The former supposes the electrical

steady state, which means that the variation of currents at estimated rotor reference frame is negligible. The latter assumes the mechanical steady state, which means acceleration is negligible. However, both assumptions do not hold in fast dynamic operating condition of SMPMSM.

This paper analyzes the problems of conventional sensorless methods, and presents a modified back-EMF observer more adequate to the highly dynamic operating conditions.

## II. BACK-EMF ESTIMATION ALGORITHM

### A. Model of SMPMSM

The stator voltage equations of a SMPMSM in a synchronous reference frame is known as following equations,

$$v_{ds}^r = R_s i_{ds}^r + L_s \frac{di_{ds}^r}{dt} - \omega_r L_s i_{qs}^r, \qquad (1)$$

$$v_{qs}^{r} = R_{s}i_{qs}^{r} + L_{s}\frac{di_{qs}^{r}}{dt} + \omega_{r}(L_{s}i_{ds}^{r} + \lambda_{f}), \qquad (2)$$

where  $R_s$  is the stator winding resistance,  $L_s$  is the stator inductance,  $v_{ds}^r$ ,  $v_{qs}^r$  are the d- and q-axes voltages,  $i_{ds}^r$ ,  $i_{qs}^r$  are the d- and q-axes currents in the rotor reference frame, and  $\omega_r$  is the rotor speed in electrical angle, and  $\lambda_f$  is the magnet flux linkage.



A sensorless control is established in the estimated rotor reference frame. The estimated rotor position might be different from the real one instantaneously or constantly according to estimation performance. Fig. 1 shows the estimated synchronous rotor reference frame and real one. The difference between real and estimated rotor position and the difference between real and estimated velocity can be defined as follows,

$$\theta_{err} \equiv \theta_r - \hat{\theta}_r \,, \tag{3}$$

$$\omega_{r\_err} \equiv \omega_r - \hat{\omega}_r = \frac{d\theta_{err}}{dt} \,. \tag{4}$$

If there is any position estimation error, the voltage equation in the estimated rotor frame can be represented as follows,

$$v_{ds}^{\hat{r}} = R_s i_{ds}^{\hat{r}} + L_s \frac{di_{ds}^r}{dt} - \hat{\omega}_r L_s i_{qs}^{\hat{r}} - \omega_r \lambda_f \sin \theta_{err}, \qquad (5)$$

$$v_{qs}^{\hat{r}} = R_s i_{qs}^{\hat{r}} + L_s \frac{di_{qs}^{\hat{r}}}{dt} + \hat{\omega}_r L_s i_{ds}^{\hat{r}} + \omega_r \lambda_f \cos \theta_{err} , \qquad (6)$$

where the symbol '^' stands for the estimated value. The two equations are almost identical except the last terms, back-EMF terms, which are caused by the position error,  $\theta_{err}$ .

#### B. Conventional Estimation Methods

Sensorless control methods based on the back-EMF estimation extract the back-EMF terms from (5) and (6) and compute the position error. In principle, there are two kinds of approaches to estimate the back-EMF information.

One method is calculating it from the voltage equations directly [1]-[3]. Assuming electrical steady state  $\left(\frac{di_{ds}^{\dot{r}}}{dt} \approx 0, \frac{di_{qs}^{\dot{r}}}{dt} \approx 0\right)$ , the EMF terms can be

obtained with following simple calculations.

$$\omega_r \lambda_f \sin \theta_{err} = v_{ds}^{\hat{r}} - R_s i_{ds}^{\hat{r}} + \hat{\omega}_r L_s i_{qs}^{\hat{r}}, \qquad (7)$$

$$\omega_r \lambda_f \cos \theta_{err} = v_{qs}^{\hat{r}} - R_s i_{qs}^{\hat{r}} - \hat{\omega}_r L_s i_{ds}^{\hat{r}} \,. \tag{8}$$

If the parameter variations can be ignored and the position error is relatively small, the position error is derived as (9).

$$\frac{\omega_r \lambda_f \sin \theta_{err}}{\hat{\omega}_r \lambda_f} \approx \sin \theta_{err} \approx \theta_{err} \cdot$$
(9)

Otherwise, the arctan function can be used from (7) and (8).

$$\theta_{err} = \tan^{-1}\left(\frac{\omega_r \lambda_f \sin \theta_{err}}{\omega_r \lambda_f \cos \theta_{err}}\right).$$
(10)

However, this method can not be applied when the time derivatives of the currents in (5) and (6) are quite large. Therefore, it is not suitable to estimate the position error by the voltage equations directly when the load and/or the speed varies rapidly.

The other method is using the state observer [4]-[7]. From (5) and (6), the state equations can be derived and the closed loop state observer can be implemented as (11) and (12). In (11), back-EMF terms are defined as (13) under the assumption that the time derivatives of the

back-EMF terms are zero 
$$\left(\frac{de_d}{dt} \approx 0, \frac{de_q}{dt} \approx 0\right)$$
.

$$\hat{x} = \hat{A}\hat{x} + Bu + L(y - \hat{y})$$

$$= \begin{bmatrix} -\frac{R_s}{L_s} & \hat{\omega}_r & \frac{1}{L_s} & 0 \\ -\hat{\omega}_r & -\frac{R_s}{L_s} & 0 & \frac{1}{L_s} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u + L(y - \hat{y})$$

$$\text{where } x = \begin{bmatrix} \hat{i}_{s}^{\hat{r}} & \hat{i}_{qs}^{\hat{r}} & \hat{e}_d & \hat{e}_q \end{bmatrix}^T, \ u = \begin{bmatrix} y_{ds}^{\hat{r}} & y_{qs}^{\hat{r}} \end{bmatrix}^T, \text{ and }$$

$$L = \begin{bmatrix} l_{11} & l_{21} & l_{31} & l_{41} \\ l_{12} & l_{22} & l_{32} & l_{42} \end{bmatrix} ,$$
(11)

$$\hat{y} = C\hat{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \hat{x} \,. \tag{12}$$

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} \equiv \begin{bmatrix} \omega_r \lambda_f \sin \theta_{err} \\ -\omega_r \lambda_f \cos \theta_{err} \end{bmatrix},$$
(13)

From the back-EMF terms estimated by the state observer, position error can be derived as (9) or (10), same as the previous method. Although the calculation sequence looks similar, the state observer is more robust to operating condition and parameter errors than the strategy using the voltage equations directly due to the consideration of the current variations and also due to the closed form of the estimation.

As mentioned before, however, the conventional state observer neglects the transient state of the back-EMF terms. This assumption may be not justified in some extreme operating conditions, where this paper is focused on. The differentiation of the time of the back-EMF terms can be expressed as (14).

$$\frac{d}{dt}\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \lambda_f \begin{bmatrix} \omega_r \cdot \omega_{r\_err} \cos \theta_{err} + \dot{\omega}_r \sin \theta_{err} \\ \omega_r \cdot \omega_{r\_err} \sin \theta_{err} - \dot{\omega}_r \cos \theta_{err} \end{bmatrix},$$
(14)

where  $\dot{\omega}_r$  is the acceleration of the rotor in electrical angle. The state equations in (11) can be modified as (15) to consider the terms in (14).

 $\dot{x} = Ax + Bu$ 

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$$= \begin{vmatrix} -\frac{R_s}{L_s} & \hat{\omega}_r & \frac{1}{L_s} & 0 \\ -\hat{\omega}_r & -\frac{R_s}{L_s} & 0 & \frac{1}{L_s} \\ 0 & 0 & \frac{\dot{\omega}_r}{\omega_r} & -\omega_{r\_err} \\ 0 & 0 & \omega_{r\_err} & \frac{\dot{\omega}_r}{\omega_r} \end{vmatrix} \hat{x} + \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$
(15)

The state equation (15) describes the dynamics of the drive system when the speed varies rapidly. And the state equation in (11) would discord with the real system when the acceleration or deceleration of the rotor is severe. Hence, the state observer by (11) reveals degraded performance or even be unstable when  $\dot{\omega}_r / \omega_r$  and/or  $\omega_{r\_err}$  are large. Fig. 2 shows the frequency analysis of back-EMF terms ( $e_d$  and  $e_q$ ) between the real model, (15), and the state observer, (11) at certain operating

condition. The transfer functions are determined by following values:  $\theta_{err}$ ,  $\dot{\omega}_r / \omega_r$ ,  $\omega_{r\_err}$ , and the poles of the state observer.

Those values are set according to the following operating condition: Rotor accelerates from 1,000 *r/min* to 3,000 *r/min* with acceleration of 200,000 (*r/min*)/s.  $\omega_{r\_err}$  is set proportional to the acceleration and  $\theta_{err}$  is set to 0.2 *rad* which is determined by simulation results. The gains of the state observer are set to place all closed-loop eigenvalues of the observer at 500 *Hz*.



Fig. 2. Transfer functions of back-EMF terms between the real model and the state observer

Each graph of Fig. 2 shows the frequency response according to different  $\dot{\omega}_r / \omega_r$  values. As the accelerating rate increases, the estimation error of  $e_d$  and  $e_q$  becomes larger. Because the sensorless algorithm calculates the  $\theta_{err}$  using them, as shown in (9) or (10), this problem is directly related to the position estimation performance. And generally, fast change of current is accompanied with a rapid acceleration, which can cause estimation error of back-EMF term instantaneously further.

Therefore, in rapid accelerating situation, both two factors, larger acceleration rate and fast change of current, would worsen the estimation performance. And if the position error exceeds a certain limit, the sensorless control might be unstable.

## C. Proposed Estimation Method

The proposed estimation method transforms state equations by defining the flux dimensional terms as (16) instead of whole back-EMF terms in (11). Then the closed-loop state observer can be configured as (17).

$$\begin{bmatrix} \lambda_{err\_d} \\ \lambda_{err\_q} \end{bmatrix} \equiv \begin{bmatrix} \lambda_f \sin \theta_{err} \\ -\lambda_f \cos \theta_{err} \end{bmatrix},$$
(16)

$$\dot{\hat{x}}' = \hat{A}\hat{x}' + Bu + L(y - \hat{y})$$

$$= \begin{bmatrix} -\frac{R_s}{L_s} & \hat{\omega}_r & \frac{\omega_r}{L_s} & 0\\ -\hat{\omega}_r & -\frac{R_s}{L_s} & 0 & \frac{\omega_r}{L_s}\\ 0 & 0 & 0 & -\omega_{r\_err}\\ 0 & 0 & \omega_{r\_err} & 0 \end{bmatrix} \hat{x}' + \begin{bmatrix} \frac{1}{L_s} & 0\\ 0 & \frac{1}{L_s}\\ 0 & 0\\ 0 & 0 \end{bmatrix} u + L(y - \hat{y})$$
where  $x' = \begin{bmatrix} \hat{l}_{ds}^{\hat{r}} & \hat{l}_{qs}^{\hat{r}} & \hat{\lambda}_{err\_d} & \hat{\lambda}_{err\_q} \end{bmatrix}^T$ . (17)

The system matrix of the proposed state equation does not contain the acceleration term which is shown in (15) by defining the flux dimensional term as state variables. Thus, the system matrix of the modified state observer does not differ from that of the real model even on rapid acceleration; therefore it is expected that the estimation performance of the proposed state observer could be better than that of the conventional one.

From the estimated flux terms, the estimated position error can be derived by using arctan function as (18).

$$\theta_{err} = \tan^{-1}\left(\frac{\hat{\lambda}_{err\_d}}{-\hat{\lambda}_{err\_q}}\right) \cdot$$
(18)

This paper uses a speed observer to estimate the position and the velocity of the rotor from the estimated position error. This observer has similar form of conventional PLL(Phase Locked Loop) and employs the torque reference as a feedforward command to enhance the tracking capability [6].



The estimated position error is fed into the speed observer, shown in Fig. 3. The outputs of the speed observer, estimated speed and angle, are used in the current regulator and the speed controller. The gains of the observer  $(l_1, l_2, l_3$  in Fig. 3) are set to locate the

closed-loop eigenvaules at a certain value.

The speed error term used in (17),  $\omega_{r\_err}$ , can be acquired in the speed observer also. The speed error occurs at transient state. At transient state, when sudden change of current and/or speed occur, estimation error of the flux terms arises instantaneously and it causes the position error, consequently. The position error occurred in this situation is consisted of high frequency components. Then it incurs the high frequency component in the estimated speed although the actual speed does not vary in such manner. Therefore, high frequency component of estimated speed at transient state can be disregarded. Thus, relations between and actual and the estimated speed can be approximated as follows,

$$\omega_{rm} \approx \hat{\omega}_{rm} \frac{\omega_c}{s + \omega_c},\tag{19}$$

$$\omega_{rm\_err} = \omega_{rm} - \hat{\omega}_{rm}$$

$$= \hat{\omega}_{rm} \left( \frac{\omega_c}{s + \omega_c} - 1 \right) = -\hat{\omega}_{rm} \frac{s}{s + \omega_c},$$
(20)

where  $\omega_{rm}$  and  $\hat{\omega}_{rm}$  are the real and estimated rotor speed in the mechanical angle and  $\omega_c$  is the filter cutoff frequency. And, the speed error can be acquired through the high-pass filtering of the estimated speed. The cutoff frequency can be set by considering the bandwidth of the speed controller.



 TABLE I

 SPECIFICATIONS AND PARAMETERS

 Number of pole pairs
 4

 Rated speed
 3000 r/min

 Rated torque
 2.55 Nm

 Bandwidth of the speed controller
 70 Hz

 Eigenvalues of the speed observer
 70 Hz

 Eigenvalues of the state observer
 500 Hz

Experimental tests are carried out to evaluate the performance of the proposed sensorless strategy and compare the performance of the proposed observer to that of the conventional observer. The overall experimental system is illustrated in Fig. 4. A SMPMSM is driven by an IGBT inverter switching at 10 kHz. And a high performance Digital Signal Processor (DSP), TMS320VC33, is used to implement overall sensorless control algorithm, including the speed and current controller. In order to estimate  $\theta_{err}$  and for the comparison, the conventional observer, (11) and the

proposed observer, (17) are both integrated to the controller. A position sensor is installed to monitor the real position and speed. These monitored values are compared with estimated ones. The specifications of SMPMSM under test and control parameters are listed on Table I.

In order to achieve rapid acceleration, the bandwidth of the speed controller should be set as high as possible. The precondition of the higher speed regulation bandwidth is the fast enough estimation of the angle and the speed of the rotor. The dynamics of the estimation process is governed by the eigenvalues of the speed observer and the state observer. However, because of the stability issues due to the delays of the signal processing and the errors in the parameters of the observer, the eigenvalues can not be raised above a threshold. This threshold can be raised in the case of the proposed observer compared to the conventional observer. However, still there is threshold. The experiment was conducted with both observers and the results are presented.

Fig. 5 presents the experimental results of the sensorless control with two different observers at a rapid acceleration. Each waveform stands for the real and estimated speed, real position error and estimated position error. The rotor speed is accelerated from 1000 r/min to 3000 r/min and the acceleration rate is set as 200,000 (r/min)/s. The position error oscillates when the rotor accelerates rapidly due to the variation of the speed and the currents. However, the extent of the oscillation is smaller with proposed observer. These results show that proposed observer is more robust than the conventional one at transient state. It means that the bandwidth of the speed control system can be extended.

Fig. 6 shows the steady state performance of the proposed strategy at 2000 r/min. Both estimated angle and rotor speed well track the real values. It can be considered that the proposed sensorless control is stable in the steady state.





Fig. 6. Sensorless performance with the proposed method under the steady state

## **IV. CONCLUSIONS**

This paper has proposed a sensorless method for SMPMSM, a modification form of conventional back-EMF estimation method. Conventional sensorless methods based on back-EMF estimation have somewhat of restrictions to estimate the position error at transient state. Such problems originate from the differences between the real model configurations and the state observer; the time derivative of current or velocity is not modeled in the conventional sensorless estimation methods, thus capability to estimate position error is limited when the current or speed is varying rapidly.

In order to overcome this performance limitation, this paper proposes the modified estimation method, which employs more accurate system model at the transient state than the conventional model. The proposed algorithm has been verified with the experimental results at not only steady state but also transient state. The proposed sensorless control method can enhance the estimation performance in some applications where fast dynamics are required.

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